

# Seeking the Shadowing in $eA$ Processes

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**Abstract:** We consider the behavior of the slope of the nuclear structure function  $F_2^A$  in the kinematic region which will be explored in the  $eA$  HERA collider. We demonstrate that, similarly to the nucleon case, a 'turn over' is predicted in this observable. Moreover, we predict that the presence of the shadowing corrections implies that the maximum value of the slope is dependent of the number of nucleons  $A$ , differently from the DGLAP predictions. Our conclusion is that the measurement of this observable will allow to evidenciate the presence of the shadowing corrections.

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# 1 Introduction

In recent years several experiments have been dedicated to high precision measurements of deep inelastic lepton scattering (DIS) off nuclei. Experiments at CERN and Fermilab focus especially on the region of small values of the Bjorken variable  $x = Q^2/2M\nu$ , where  $Q^2 = -q^2$  is the squared four-momentum transfer,  $\nu$  the energy transfer and  $M$  the nucleon mass. The data [1], taken over a wide kinematic range  $10^{-5} \leq x \leq 0.1$  and  $0.05 \text{ GeV}^2 \leq Q^2 \leq 100 \text{ GeV}^2$ , show a systematic reduction of the nuclear structure function  $F_2^A(x, Q^2)/A$  with respect to the free nucleon structure function  $F_2^N(x, Q^2)$ . This phenomena is known as *the shadowing effect*. The analysis of the shadowing corrections for the nuclear case in deep inelastic scattering (DIS) has been extensively discussed [2]. It is motivated by the perspective that in a near future an experimental investigation of the nuclear shadowing at small  $x$  and  $Q^2 \gg 1 \text{ GeV}^2$  using  $eA$  scattering could occur at DESY Hadron Electron Ring Accelerator (HERA). Measurements over the extended  $x$  and  $Q^2$  ranges, which would become possible at HERA, will give more information in order to discriminate between the distinct models of shadowing and the understanding of the phenomenon which limits the rise of the proton structure function  $F_2$  at small  $x$ .

The deep inelastic scattering off a nucleus is usually interpreted in a frame where the nucleus is going very fast. In this case the nuclear shadowing is a result of an overlap in the longitudinal direction of the parton clouds originated from different bound nucleons [3]. It corresponds to the fact that small  $x$  partons cannot be localized longitudinally to better than the size of the nucleus. Thus low  $x$  partons from different nucleons overlap spatially creating much larger parton densities than in the free nucleon case. This leads to a large amplification of the nonlinear effects expected in QCD at small  $x$ . In the target rest frame, the electron-nucleus scattering at HERA allows a new regime to be probed experimentally for the first time. This is a new regime in which the virtual photon interacts coherently with all the nucleons at a given impact parameter. This can be visualized in terms of the propagation of a small  $q\bar{q}$  pair in high density gluon fields through much larger distances than it is possible with free nucleons. Few years ago, a perturbative approach has been developed to calculate the gluon distribution in a nucleus [4] using perturbative QCD at small  $x$ . This approach, known as Glauber-Mueller (GM) approach is formulated in the target rest frame, takes

into account the fluctuations of the hard probe. It includes the shadowing corrections (SC) due to parton rescatterings inside the nucleus, and provides the SC to the nuclear gluon distribution using the solution of the DGLAP evolution equations [5] to the nucleon case. As a result the behavior of related observables ( $F_2^A, dF_2^A/d\log Q^2, F_L^A, \dots$ ) at high energies can be calculated.

The GM approach was extended for the nucleon case in [6] and a comprehensive phenomenological analysis of the behavior of distinct observables ( $F_2, F_L, F_2^c$ ) was made for the  $ep$  HERA kinematical region using this approach [7]. Our main conclusion was that the unitarity corrections are large in the HERA kinematical region, but only new data, with better statistics, will allow to discriminate these corrections from the DGLAP predictions. The recent ZEUS data [8] for the slope of the proton structure function presents a 'turn over' which cannot be reproduced by the DGLAP evolution equations with the GRV95 parameterization [9]. Initially, this behavior was interpreted as the first evidence of the shadowing corrections in the kinematic region of the  $ep$  HERA collider [10, 11]. However, the MRST [12] and GRV [13] groups have produced a new set of parameterizations of the parton distributions which also reproduced the data. Therefore the current situation of the  $ep$  HERA data still cannot demonstrate clearly the presence of the shadowing corrections. This conclusion motivates an analysis of these corrections in other processes. In this Letter we analyze the  $A$  dependence of the slope of the nuclear structure function which should be measured in the future in the  $eA$  HERA collider.

Lets us start from the space-time picture of the  $eA$  processes [14]. The deep inelastic scattering  $eA \rightarrow e + X$  is characterized by a large electron energy loss  $\nu$  (in the target rest frame) and an invariant momentum transfer  $q^2 \equiv -Q^2$  between the incoming and outgoing electron such that  $x = Q^2/2m_N\nu$  is fixed. The general features of the time development can be established using only Lorentz invariance and the uncertainty principle. The incoming physical electron state can, at a given instant of time, be expanded in terms of its (bare) Fock states

$$|e>_{phys} = \psi_e|e> + \psi_{e\gamma}|e\gamma> + \dots \quad (1)$$

The amplitudes  $\psi_i$  depend on the kinematic variables describing the states  $|i>$ , and have the time dependence  $\exp(-iE_it)$ , where  $E_i = \sum_i \sqrt{m_i^2 + \vec{p}_i^2}$  is the energy of the state. The 'lifetime'  $\tau_i \approx 1/(E_i - E_e)$  of a Fock state  $|i>$  is

given by the time interval after which the relative phase  $\exp[-i(E_i - E_e)]$  is significantly different from unity. If  $\tau_i > R_A$  the Fock state forms long before the electron arrives at the nucleus, and it lives long after its passage. New Fock states are not formed inside the nucleus. Therefore, the scattering inside the nucleus is diagonal in the Fock basis. If the state  $|i\rangle$  contains particles with mass  $m_j$ , energy fraction  $x_j$  and transverse momentum  $p_{tj}$ , we have that the transverse velocities  $v_{tj} = p_{tj}/x_j E_e$  are small at large  $E_e$ . Hence the impact parameters (transverse coordinates) of all particles are preserved.

In terms of Fock states we then view the  $eA$  scattering as follows: the electron emits a photon ( $|e\rangle \rightarrow |e\gamma\rangle$ ) with  $E_\gamma = \nu$  and  $p_{t\gamma}^2 \approx Q^2$ , after the photon splits into a  $q\bar{q}$  ( $|e\gamma\rangle \rightarrow |eq\bar{q}\rangle$ ) and typically travels a distance  $l_c \approx 1/m_N x$ , referred as the 'coherence length', before interacting in the nucleus. For small  $x$ , the photon converts to a quark pair at a large distance before it interacts to the target; for example, at the  $ep$  HERA collider, where one can study structure functions at  $x \approx 10^{-5}$ , the coherence length is as large as  $10^4 fm$ , much larger than the nuclear radii. Consequently, the space-time picture of the DIS in the target rest frame can be viewed as the decay of the virtual photon at high energy (small  $x$ ) into a quark-antiquark pair long before the interaction with the target. The  $q\bar{q}$  pair subsequently interacts with the target. In the small  $x$  region, where  $x \ll \frac{1}{2m_R}$ , the  $q\bar{q}$  pair crosses the target with fixed transverse distance  $r_t$  between the quarks. It allows to factorize the total cross section between the wave function of the photon and the interaction cross section of the quark-antiquark pair with the target. The photon wave function is calculable and the interaction cross section is modelled. Therefore, the nuclear structure function is given by [15]

$$F_2^A(x, Q^2) = \frac{Q^2}{4\pi\alpha_{em}} \int dz \int \frac{d^2 r_t}{\pi} |\Psi(z, r_t)|^2 \sigma^{q\bar{q}+A}(z, r_t), \quad (2)$$

where

$$|\Psi(z, r_t)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_i^{n_f} e_f^2 \{ [z^2 + (1-z)^2] \epsilon^2 K_1(\epsilon r_t)^2 + m_f^2 K_0^2(\epsilon r_t)^2 \}, \quad (3)$$

$\alpha_{em}$  is the electromagnetic coupling constant,  $\epsilon = z(1-z)Q^2 + m_f^2$ ,  $m_f$  is the quark mass,  $n_f$  is the number of active flavors,  $e_f^2$  is the square of the parton charge (in units of  $e$ ),  $K_{0,1}$  are the modified Bessel functions and  $z$  is the fraction of the photon's light-cone momentum carried by one of the

quarks of the pair. In the leading  $\log(1/x)$  approximation we can neglect the change of  $z$  during the interaction and describe the cross section  $\sigma^{q\bar{q}+A}(z, r_t^2)$  as a function of the variable  $x$ .

We estimated the unitarity corrections considering the Glauber multiple scattering theory [16], which was probed in QCD [17]. The nuclear collision is analysed as a succession of collisions of the probe with individual nucleons within the nucleus, and summarizing we obtain that the  $F_2$  structure function can be written as [4, 18]

$$F_2^A(x, Q^2) = \frac{R_A^2}{2\pi^2} \sum_1^{n_f} \epsilon_i^2 \int_{\frac{1}{Q^2}}^{\frac{1}{Q_0^2}} \frac{d^2 r_t}{\pi r_t^4} \{C + \ln(\kappa_q(x, r_t^2)) + E_1(\kappa_q(x, r_t^2))\} , \quad (4)$$

where  $A$  is the number of nucleons,  $R_A^2$  is the mean nuclear radius and  $\kappa_q = (2\alpha_s A/3R_A^2) \pi r_t^2 x G_N(x, \frac{1}{r_t^2})$  (see [18] for details).

The slope of the nuclear structure function can be obtained directly from the expression (4). We obtain that

$$\frac{dF_2^A(x, Q^2)}{d\log Q^2} = \frac{R_A^2 Q^2}{2\pi^2} \sum_1^{n_f} \epsilon_i^2 \{C + \ln(\kappa_q(x, Q^2)) + E_1(\kappa_q(x, Q^2))\} , \quad (5)$$

which predicts the  $x$ ,  $Q^2$  and  $A$  dependence of the shadowing corrections for the  $F_2^A$  slope.

We see that the behavior of the  $F_2^A$  and its slope are strongly dependent on the nucleon gluon distribution. This is a common characteristic of the observables in the small  $x$  region, where the gluon distribution dominates. Therefore, before we make predictions for these observables an analysis of the behavior of the gluon distribution should be made. In the nucleon case, a strong growth of the nucleon structure function is observed in the  $ep$  HERA data, which violates the unitarity boundary [19]. Consequently, we expect that unitarity corrections will be present in the  $ep$  HERA kinematical region. In [11] we have shown that the ZEUS data for the  $F_2^p$  slope [8] which presents a turn over can only be successfully described considering the expression (5) for the nucleon case and a shadowed gluon distribution (quark + gluon sectors). In this Letter we consider that the nucleon gluon distribution is calculated using the GM approach, *i. e.* we consider that the behavior of the gluon distribution was modified by the unitarity corrections (see [11] for details).

The behavior of the nuclear structure function was analysed in [18] using the Glauber-Mueller approach. We have shown that the ratio  $R_1 = F_2^A/(AF_2^p)$  is strongly modified by the shadowing corrections and that it saturates in the perturbative regime ( $Q^2 \geq 1 \text{ GeV}^2$ ) when both the quark and gluon sectors are considered. Here we estimate the shadowing corrections for the  $F_2^A$  slope in the HERA kinematic region, where  $s = 9 \times 10^4 \text{ GeV}^2$ . Following [8], where the data points correspond to different  $x$  and  $Q^2$ , we consider that the variables  $x$  and  $Q^2$  are related by the expression  $x = Q^2/(sy)$  and that the inelasticity variable  $y$  is given by  $y = 0.25$ . This is a typical value in the measurements at HERA [20].

In figure (1) we present our predictions (solid curve) for the behavior of the  $F_2^A$  slope using the expression (5). We compare our results with the predictions of the DGLAP evolution equations using the GRV parameterization (dashed curve) without any nuclear effect. We see that a 'turn over' is present in the DGLAP (GRV) and GM predictions. However, the remarkable property of the result is that the shadowing corrections modify the maximum for each of the slopes and that it is  $A$  dependent. This is expected from the formalism based on GM approach as extensively explained in [4]. In table (1) we present explicitly the  $A$  dependence of the 'turn over' in the  $F_2^A$  slope. In the Calcium case ( $A = 40$ ) we predict that the 'turn over' occurs at  $Q^2 = 5 \text{ GeV}^2$ , differently from the DGLAP (GRV) case for which is at  $Q^2 = 1.7 \text{ GeV}^2$  independently of  $A$ . We believe that this behavior cannot be mimicked by modifications in the parton parameterizations, which makes  $dF_2^A/d\log Q^2$  a sensitive probe of the shadowing corrections.

The behavior of the  $F_2^A$  slope can be understood intuitively. The  $A$  dependence of the 'turn over' is associated with the regime in which the partons in the nucleus form a dense system with mutual interactions and recombinations. The recombinations, *i.e* the shadowing corrections, occur predominantly at large density. As the partonic density growth at larger values of the number of nucleons  $A$  and smaller values of  $x$ , the same density at  $A = 1, 40, 197$  is obtained at larger values of  $x$  and in extension  $Q^2$ . This behavior of the recombinations is verified in the  $F_2^A$  slope.

Our main conclusion is that the analysis of the slope of the nuclear structure functions at  $eA$  HERA energies will allow to discriminate the presence of the shadowing corrections from the DGLAP predictions. Our result has important implications in the nucleon case and in QCD at high densities. In the nucleon case, the evidence of an  $A$  dependence of the 'turn over'

will demonstrate that the correct way to estimate the observables in the  $ep$  HERA collider is considering the shadowing corrections, without modifying the parton distributions. On the other hand, in the near future, the collider facilities such as BNL Relativistic Heavy Ion Collider (RHIC), and CERN Large Hadron Collider (LHC) ( $p\bar{p}$ ,  $AA$ ) will be able to probe new regimes of dense quark matter at very small Bjorken  $x$  or/and at large  $A$ , with rather different dynamical properties. The description of these processes is directly associated with a correct description of the dynamics of minijet production, which will be strongly modified by shadowing corrections. We expect that this result contributes to motivate the running of nucleus at HERA in the future.

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## Tables

	GRV $x$	$Q^2 (GeV^2)$	GM $x$	$Q^2 (GeV^2)$
$A = 1$	$0.75 \times 10^{-4}$	1.7	$0.48 \times 10^{-4}$	1.1
$A = 40$	$0.75 \times 10^{-4}$	1.7	$0.22 \times 10^{-3}$	5.0
$A = 197$	$0.75 \times 10^{-4}$	1.7	$0.28 \times 10^{-3}$	6.5

Table 1: The  $A$  dependence of the 'turn over' predicted in the  $F_2^A$  slope.

## Figure Captions

Fig. 1: The  $F_2^A$  slope as function of the variable  $x$  at different values of  $A$ . Each value of  $x$  is related with the virtuality  $Q^2$  by the expression  $x = Q^2/(sy)$ , where we have assumed  $s = 9 \times 10^4 \text{ GeV}^2$  and  $y = 0.25$ . See text.

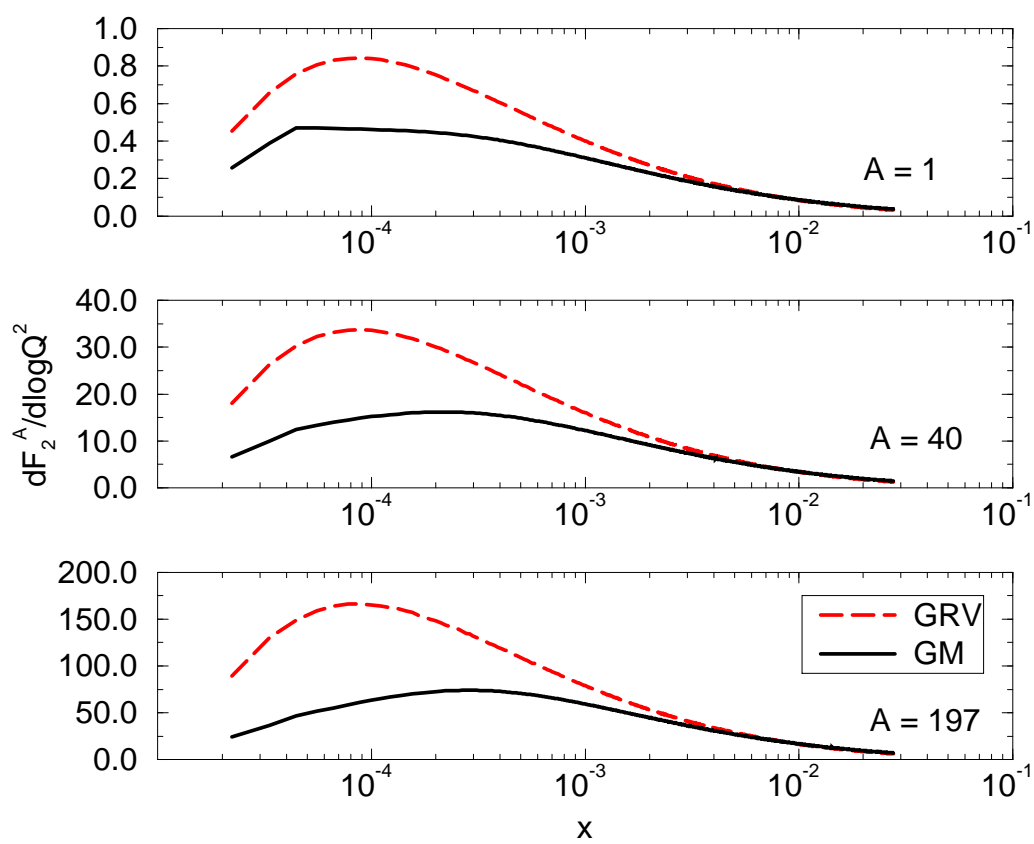


Figure 1: